1.3 More on Functions and Their Graphs

**Objectives**

1. Identify intervals on which a function increases, decreases, or is constant.
2. Use graphs to locate relative maximums or minimums.
3. Identify even or odd functions and recognize their symmetries.
4. Understand and use piecewise functions.
5. Find and simplify a function's difference quotient.

**Increasing, Decreasing, and Constant Functions**

- A function is increasing on an open interval I if \( f(x_2) > f(x_1) \) whenever \( x_2 \gt x_1 \) for any \( x_1 \) and \( x_2 \) in the interval.
- A function is decreasing on an open interval I if \( f(x_2) < f(x_1) \) whenever \( x_2 \lt x_1 \) for any \( x_1 \) and \( x_2 \) in the interval.

**Relative Maxima and Relative Minima**

The points at which a function changes its increasing or decreasing behavior can be used to find the relative maximums or relative minimums values of the function.

**Definitions of Relative Maximum and Relative Minimum**

1. A function value \( f(c) \) is a relative maximum of \( f \) if there exists an open interval containing \( c \) such that \( f(c) \geq f(x) \) for all \( x \) in the open interval.
2. A function value \( f(c) \) is a relative minimum of \( f \) if there exists an open interval containing \( c \) such that \( f(c) \leq f(x) \) for all \( x \) in the open interval.
Find the numbers at which \( f \) has relative maximum/minima.

What are these relative maxima?

\[
\begin{align*}
\text{Relative Maxima} & : (152, 37) \\
\text{Relative Minima} & : (-27, 37)
\end{align*}
\]

Definitions of Even and Odd Functions

The function \( f \) is an even function if \( f(-x) = f(x) \) for all \( x \) in the domain of \( f \).

The right side of the equation of an even function does not change if \( x \) is replaced with \(-x\).

The function \( f \) is an odd function if \( f(-x) = -f(x) \) for all \( x \) in the domain of \( f \).

Every term on the right side of the equation of an odd function changes its sign if \( x \) is replaced with \(-x\).

Even Functions and \( y \)-Axis Symmetry

The graph of an even function is symmetric with respect to the \( y \)-axis.

Odd Functions and Origin Symmetry

The graph of an odd function is symmetric with respect to the origin.

State whether the function is odd, even or neither. Check graphically. Confirm algebraically.

\[
\begin{align*}
f(x) &= 2x^2 - 3x \\
f(-x) &= 2(-x)^2 - 3(-x) \\
&= 2x^2 + 3x \\
&= -(2x^2 - 3x) = \text{odd}
\end{align*}
\]

\[
\begin{align*}
f(x) &= -x^2 + 0.3x + 5 \\
f(-x) &= -(x)^2 + 0.3(-x) + 5 \\
&= -x^2 - 0.3x + 5 = \text{odd}
\end{align*}
\]

Piecewise functions:

\[
\begin{align*}
f(x) &= \begin{cases} 
1 & x \leq -1 \\
x^2 - 15x + 1 & -1 < x \leq 1 \\
x & x \geq 1
\end{cases}
\end{align*}
\]

Domain: \((-\infty, \infty)\)

Range: \([-10, \infty)\)

Increasing Interval: \((-1, 1)\)

Decreasing Interval: \((-\infty, -1)\)

Constant Interval: \((-\infty, -1)\)

Local Max: \(0\)

Local Min: \(-10\)
Piecewise functions:

\[
f(x) = \begin{cases} 
  x + 2 & x < -2 \\
  x & -2 \leq x < 0 \\
  2 & x \geq 0
\end{cases}
\]

Domain: \((-\infty, \infty)\)
Range: \([-2, 0) \cup (0, \infty)\)

Inc: \((-\infty, -2)\)
Dec: \((-2, 0)\)
Constant: \((0, \infty)\)

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**Functions and Difference Quotients**

In this section, we will be studying the average rate of change of a function. A rate called the difference quotient plays an important role in understanding the rate at which functions change.

**Definition of the Difference Quotient of a Function**

The expression

\[
\frac{f(x+h) - f(x)}{h}
\]

for \(h \neq 0\) is called the difference quotient of the function \(f\).

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**Assignment**

Lesson 1.3

1, 7, 9, 13, 15, 17-21 odd (alg), 29-33, 51, 53