**Modeling Periodic Behavior**

**Example 8** A Trigonometric Breath of Life

The graph in Figure 4.76 shows one complete normal breathing cycle. The cycle consists of inhaling and exhaling. It takes place every 5 seconds. Velocity of air flow is positive when we inhale and negative when we exhale. It is measured in liters per second. If $y$ represents velocity of air flow after $x$ seconds, find a function of the form $y = A \sin Bx$ that models air flow in a normal breathing cycle.

\[
y = A \sin Bx
\]

Using $y = .6 \sin (\frac{2\pi}{5}x)$

\[
S = \frac{2\pi}{B}
\]

\[
B = \frac{2\pi}{S} = 5\text{ seconds}
\]

**Modeling a Tidal Cycle**

Figure 4.77 shows that the depth of water at a boat dock varies with the tides. The depth is 5 feet at low tide and 13 feet at high tide. On a certain day, low tide occurs at 4 A.M. and high tide at 10 A.M. If $y$ represents the depth of the water, in feet, $x$ hours after midnight, use a sine function of the form $y = A \sin(Bx - C) + D$ to model the water's depth.

\[
y = A \sin(Bx - C) + D
\]

\[
y = 4 \sin \left( \frac{\pi}{6}x - \frac{2\pi}{3} \right) + 9
\]

**Making a Tidal Cycle**

\[
\text{Per} = \frac{2\pi}{B}
\]

\[
p \cdot s = \frac{C}{B}
\]

\[
x = \frac{2\pi}{B} \quad B = \frac{\pi}{6} \quad 7 = \frac{C}{\frac{\pi}{6}} \quad C = 7\pi
\]
Hours of Daylight

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let $x$ represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If $y$ represents the number of hours of daylight in month $x$, use a sine function of the form $y = A \sin(Bx - C) + D$ to model the hours of daylight.

\[
y = 2 \sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12
\]

\[
\text{Per} = \frac{2\pi}{B} \quad B = \frac{\pi}{6} \\
\text{Phase Shift} = \frac{C}{B} = \frac{\pi}{2} \\
\text{Amplitude} = A = \frac{2}{2} = 1 \\
\text{Vertical Shift} = D = 12
\]

\[
f(x) = 2 \sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12
\]

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$y = \tan x$

http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html

- The period is $\pi$. It is only necessary to graph $y = \tan x$ over an interval of length $\pi$. The remainder of the graph consists of repetitions of that graph at intervals of $\pi$.
- The tangent function is an odd function: $\tan(-x) = -\tan x$. The graph is symmetric with respect to the origin.
- The tangent function is undefined at $\frac{\pi}{2}$. The graph of $y = \tan x$ has a vertical asymptote at $x = \frac{\pi}{2}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{4}$</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{5\pi}{12}$ (75°)</th>
<th>$\frac{\pi}{2}$ (90°)</th>
<th>$\frac{7\pi}{12}$ (105°)</th>
<th>$\frac{\pi}{2}$ (180°)</th>
<th>$\frac{9\pi}{8}$ (150°)</th>
<th>$\frac{11\pi}{6}$ (165°)</th>
<th>$\frac{3\pi}{2}$</th>
<th>$\frac{5\pi}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \tan x$</td>
<td>0</td>
<td>$\sqrt{3}$ ≈ 0.6</td>
<td>1</td>
<td>$\sqrt{3}$ ≈ 1.7</td>
<td>3.7</td>
<td>12.55</td>
<td>12.55</td>
<td>12.55</td>
<td>12.55</td>
<td>12.55</td>
<td>12.55</td>
<td>12.55</td>
</tr>
</tbody>
</table>

As $x$ increases from 0 toward $\frac{\pi}{2}$, $y$ increases slowly at first, then more and more rapidly.
The Tangent Curve: The Graph of $y = \tan x$ and Its Characteristics

**Characteristics**

- **Period:** $\pi$
- **Domain:** All real numbers except odd multiples of $\frac{\pi}{2}$
- **Range:** All real numbers
- **Vertical asymptotes** at odd multiples of $\frac{\pi}{2}$
- **An x-intercept** occurs midway between each pair of consecutive asymptotes.
- **Odd function** with origin symmetry

(b) $y = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$
Graphing $y = A \tan(Bx - C)$

1. Find two consecutive asymptotes by finding an interval containing one period:
   \[ \frac{-\pi}{2} < Bx - C < \frac{\pi}{2}. \]
   A pair of consecutive asymptotes occur at
   \[ Bx - C = \frac{-\pi}{2} \text{ and } Bx - C = \frac{\pi}{2}. \]

2. Identify an $x$-intercept, midway between the consecutive asymptotes.

3. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have $y$-coordinates of $-A$ and $A$, respectively.

Graph 2 periods

$y = 2 \tan \frac{x}{2}$

Parent asymptotes:

$-\frac{\pi}{2}, \frac{\pi}{2}$

New asymptotes:

$-\pi, \pi$
Graph 2 periods \( f(x) = 3\tan 4x \)

- Parent asymptotes: \(-\frac{\pi}{2}, \frac{\pi}{2}\)
- New asymptotes: \(-\frac{\pi}{8}, \frac{\pi}{8}\)

Graph 2 periods \( y = \tan \left( x + \frac{\pi}{4} \right) \)

- Parent asymptotes: \(-\frac{\pi}{2}, \frac{\pi}{2}\)
- New asymptotes: \(-\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}-\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}\)
The Cotangent Curve: The Graph of $y = \cot x$ and Its Characteristics

Characteristics

- **Period:** $\pi$
- **Domain:** All real numbers except integral multiples of $\pi$
- **Range:** All real numbers
- **Vertical asymptotes** at integral multiples of $\pi$
- **An $x$-intercept** occurs midway between each pair of consecutive asymptotes.
- **Odd function** with origin symmetry
- Points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between consecutive asymptotes have $y$-coordinates of 1 and $-1$, respectively.

Graphing $y = A \cot(Bx - C)$

1. Find two consecutive asymptotes by finding an interval containing one full period:
   \[0 < Bx - C < \pi.\]
   A pair of consecutive asymptotes occur at
   \[Bx - C = 0\text{ and }Bx - C = \pi.\]
2. Identify an $x$-intercept, midway between the consecutive asymptotes.
3. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have $y$-coordinates of $A$ and $-A$, respectively.
$y = 3 \cot 2x$

Parent asymp:

\[
\begin{align*}
0 & \quad \frac{\pi}{2} \\
\div 2 & \\
\text{new asymp} & \quad \frac{0}{2} \quad \frac{\pi}{2}
\end{align*}
\]

Graph

$y = \frac{1}{2} \cot \frac{\pi}{2} x$

Parent asymp:

\[
\begin{align*}
0 & \quad \frac{\pi}{2} \\
\div \frac{\pi}{2} & \\
\text{new asymp} & \quad \frac{0}{2} \quad \frac{\pi}{2} \\
& \quad \frac{\pi}{2} \cdot \frac{\pi}{2} = 2
\end{align*}
\]
4.5 page 534 (75-81 odd, 85-87, 109b)

4.6 page 546 (1-5, 9, 11, 13-16, 19, 23)

Study your unit circle!!!