5.5 Solving Trigonometric Equations

Objectives
1. Find all solutions of a trigonometric equation.
2. Solve equations with multiple angles.
3. Solve trigonometric equations quadratic in form.
4. Use factoring to separate different functions in trigonometric equations.
5. Use identities to solve trigonometric equations.
6. Use a calculator to solve trigonometric equations.

Figure 5.7 The equation \( \sin x = \frac{1}{2} \) has five solutions when \( x \) is restricted to the interval \( \left[ -\frac{3\pi}{2}, \frac{7\pi}{2} \right] \).

How do we represent ALL solutions to \( \sin x = \frac{1}{2} \)?

Find all solutions to \( \sin x = \frac{1}{2} \)

\[ \sin^{-1} \frac{1}{2} = \frac{\pi}{6} \]

\[ \frac{\pi}{6} + 2\pi n \]

Where \( n \) is an integer.
Find all solutions to

\[ 4 \tan x = \sqrt{3} + \tan x \]

\[-\tan x \quad -\tan x \]

\[ \frac{3 \tan x}{3} = \frac{\sqrt{3}}{3} \]

\[ \tan x = \frac{\sqrt{3}}{3} \]

\[ \tan^{-1} \frac{\sqrt{3}}{3} = x \]

\[ \frac{\pi}{6} + n\pi \]

\[ \frac{7\pi}{6} + n\pi \]

Solve on the interval \([0, 2\pi)\)

\[ \sin x - \cos x = 0 \]

\[ \cos x \quad \cos x \]

\[ \sin x = \cos x \]

\[ \frac{\pi}{4} \quad \frac{5\pi}{4} \]

\[ \sin x \tan x = \sin x \]

\[ -\sin x \quad -\sin x \]

\[ \sin x \tan x - \sin x = 0 \]

\[ \sin x (\tan x - 1) = 0 \]

\[ \sin x = 0 \text{ or } \tan x - 1 = 0 \]

\[ x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4} \]
Equations with a Multiple Angle.

Find all solutions to

\[ 2 \sin \frac{x}{2} = -1 \]

\[ +2\pi \times 2 = +4\pi \]

\[ \sin \frac{x}{2} = -\frac{1}{2} \]

\[ \sin^{-1} -\frac{1}{2} = \frac{x}{2} \]

\[ \frac{7\pi}{6} = \frac{x}{2} \]

\[ \frac{11\pi}{6} = \frac{x}{2} \]

\[ \frac{14\pi}{6} \quad \frac{22\pi}{6} \]

\[ \frac{7\pi}{3} + 4n\pi \quad \frac{13\pi}{3} + 4n\pi \]

Solve on the interval \([0, 2\pi)\)

\[ \sin 4x = -\frac{\sqrt{2}}{2} \]

\[ +\frac{2\pi}{4} + \frac{\pi}{2} = \frac{8\pi}{6} \]

\[ \sin^{-1} -\frac{\sqrt{2}}{2} = 4x \]

\[ \frac{5\pi}{4} = 4x \quad \frac{7\pi}{4} = 4x \]

\[ +\frac{8\pi}{10} \]

\[ \frac{3\pi}{16} \quad \frac{1\pi}{16} \quad \frac{15\pi}{16} \quad \frac{23\pi}{16} \quad \frac{5\pi}{16} \]

\[ \sin 3x \cos 2x + \cos 3x \sin 2x = 1 \]

\[ \sin (3x+2x) = 1 \]

\[ \sin 5x = 1 \]

\[ +\frac{2\pi}{5} = \frac{4\pi}{10} \]

\[ \sin^{-1} 1 = 5x \]

\[ +\frac{\pi}{2} = 5x \]

\[ +\frac{8\pi}{10} \]

\[ \frac{3\pi}{10} \quad \frac{17\pi}{10} \]

\[ \frac{9\pi}{10} \quad \frac{13\pi}{10} \]
Solving Trig Equations Quadratic in Form
Solve on the interval \([0, 2\pi)\)

\[2 \cos^2 x + \cos x - 1 = 0\]

\[(2\cos x - 1)(\cos x + 1) = 0\]

\[2 \cos x - 1 = 0 \quad \text{or} \quad \cos x + 1 = 0\]

\[\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -1\]

\[\frac{\pi}{3}, \quad \frac{5\pi}{3}, \quad \pi\]

**Graph**

Using Identities to Solve Trigonometric Equations
Solve on the interval \([0, 2\pi)\)

\[\cos 2x = \cos x\]

\[\cos 2x - \cos x = 0\]

\[2 \cos^2 x - 1 - \cos x = 0\]

\[2 \cos^2 x - \cos x - 1 = 0\]

\[(2\cos x + 1)(\cos x - 1) = 0\]

\[2 \cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0\]

\[\cos x = -\frac{1}{2} \quad \cos x = 1\]

\[\frac{2\pi}{3}, \quad \frac{4\pi}{3}, \quad 0\]

**Study Tip**

In solving

\[\tan x \sin^2 x = 3 \tan x,\]

do not begin by dividing both sides by \(\tan x\). Division by zero is undefined. If you divide by \(\tan x\), you lose the two solutions for which \(\tan x = 0\), namely 0 and \(\pi\).
Using Identities to Solve Trigonometric Equations

Solve on the interval \([0, 2\pi]\)

\[
4 \sin x \cos x = \sqrt{3}
\]

\[
\Rightarrow \frac{2 \sin x \cos x}{2} = \frac{\sqrt{3}}{2}
\]

\[
2 \sin x \cos x = \frac{\sqrt{3}}{2}
\]

\[
\sin 2x = \frac{\sqrt{3}}{2}
\]

\[
\Rightarrow x = \frac{\pi}{3} + \frac{2\pi}{2} + n \pi
\]

\[
\sin^{-1} \left( \frac{\sqrt{3}}{2} \right) = 2x
\]

\[
\frac{\pi}{3} = 2x
\]

\[
2 \cdot \frac{\pi}{3} = 2x
\]

\[
\frac{\pi}{3} = x
\]

\[
\frac{7\pi}{6}
\]

\[
\frac{11\pi}{6}
\]

\[
\frac{13\pi}{6}
\]

Homework

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17-21 odd
27-31 odd
41, 47-51 odd
59, 63, 71-75 odd
Using a Calculator to Solve Trigonometric Equations

Solve on the interval \([0, 2\pi]\)

\[8 \sin^2 x - 1 = 0\]

Using a Calculator to Solve Trigonometric Equations

Solve on the interval \([0, 2\pi]\)

\[3 \tan^2 x - \tan x - 2 = 0\]
Using a Calculator to Solve Trigonometric Equations

Solve on the interval \([0, 2\pi]\)

\[3\cos^2 x + 2\cos x - 1 = 0\]

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**Homework**  
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81, 85-89 odd,  
97, 107,  
115, 123, 125  
129, 131

**Review** page 640  
1, 2, 5, 6, 9  
15, 17, 21, 23  
25, 26, 27, 29  
35 a-d, 40  
50, 53, 55, 57, 59  
61, 62, 65, 66  
68, 69