A unit vector is defined to be a vector whose magnitude is one. In many applications of vectors, it is helpful to find the unit vector that has the same direction as a given vector.

Finding the Unit Vector that Has the Same Direction as a Given Nonzero Vector $\mathbf{v}$

For any nonzero vector $\mathbf{v}$, the vector

$$\frac{\mathbf{v}}{||\mathbf{v}||}$$

is the unit vector that has the same direction as $\mathbf{v}$. To find this vector, divide $\mathbf{v}$ by its magnitude.
Find the unit vector in the same direction as $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$. Then verify that the vector has magnitude 1.

$$
\|\mathbf{v}\| = \sqrt{(5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13
$$

$$
\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{5\mathbf{i} - 12\mathbf{j}}{13} = \frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{j}
$$

$$
\left(\frac{5}{13}\right)^2 + \left(-\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169} = \frac{169}{169} = 1
$$

Consider the vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. The components $a$ and $b$ can be expressed in terms of the magnitude of $\mathbf{v}$ and the angle $\theta$ that $\mathbf{v}$ makes with the positive $x$-axis. This angle is called the direction angle of $\mathbf{v}$ and is shown in Figure 6.59. By the definitions of sine and cosine, we have

$$
\cos \theta = \frac{a}{\|\mathbf{v}\|} \quad \text{and} \quad \sin \theta = \frac{b}{\|\mathbf{v}\|}
$$

Thus,

$$
a = \|\mathbf{v}\| \cos \theta \quad \text{and} \quad b = \|\mathbf{v}\| \sin \theta.
$$

**Writing a Vector in Terms of Its Magnitude and Direction**

Let $\mathbf{v}$ be a nonzero vector. If $\theta$ is the direction angle measured from the positive $x$-axis to $\mathbf{v}$, then the vector can be expressed in terms of its magnitude and direction angle as

$$
\mathbf{v} = \|\mathbf{v}\| \cos \theta \mathbf{i} + \|\mathbf{v}\| \sin \theta \mathbf{j}.
$$
A vector that represents the direction and speed of an object in motion is called a velocity vector.

The wind is blowing at 20 miles per hour in the direction N30°W. Express its velocity as a vector in terms of \( \mathbf{i} \) and \( \mathbf{j} \).

**Solution** The vector \( \mathbf{v} \) is shown in Figure 6.60. The vector’s direction angle, from the positive x-axis to \( \mathbf{v} \), is

\[
\theta = 90° + 30° = 120°.
\]

Because the wind is blowing at 20 miles per hour, the magnitude of \( \mathbf{v} \) is 20 miles per hour: \(|\mathbf{v}| = 20\). Thus,

\[
\mathbf{v} = |\mathbf{v}| \cos \theta \mathbf{i} + |\mathbf{v}| \sin \theta \mathbf{j}
\]

Use the formula for a vector in terms of magnitude and direction.

\[
= 20 \cos 120° \mathbf{i} + 20 \sin 120° \mathbf{j}
\]

\[
= 20 \left(-\frac{1}{2}\right) \mathbf{i} + 20 \left(\frac{\sqrt{3}}{2}\right) \mathbf{j}
\]

\[
= -10 \mathbf{i} + 10 \sqrt{3} \mathbf{j}
\]

Simplify.

The wind’s velocity can be expressed in terms of \( \mathbf{i} \) and \( \mathbf{j} \) as \( \mathbf{v} = -10 \mathbf{i} + 10 \sqrt{3} \mathbf{j} \).

Find the components of a vector that is described in terms of magnitude and direction.

The wind is blowing at 5 mph in a direction N40°W. Express its velocity as a vector \( \mathbf{v} \) in terms of \( \mathbf{i} \) and \( \mathbf{j} \).

\[
\mathbf{v} = 5 \cos 130° \mathbf{i} + 5 \sin 130° \mathbf{j}
\]

\[
\mathbf{v} = -3.21 \mathbf{i} + 3.83 \mathbf{j}
\]
A plane is flying at N65°E with a speed of 500 mph. Express its velocity as a vector in terms of \( i \) and \( j \).

\[
\vec{v} = 500 \cos 25° i + 500 \sin 25° j
\]

\[
\vec{v} = 453.15 i + 211.31 j
\]

Two forces, \( F_1 \) and \( F_2 \), of magnitude 10 and 30 pounds, respectively, act on an object. The direction of \( F_1 \) is N20°E and the direction of \( F_2 \) is N65°E. Find the magnitude and the direction of the resultant force. Express the magnitude to the nearest hundredth of a pound and the direction angle to the nearest tenth of a degree.

\[
F_1 = 10 \cos 70° i + 10 \sin 70° j = 3.42 i + 9.40 j
\]

\[
F_2 = 30 \cos 25° i + 30 \sin 25° j = 27.19 i + 12.8 j
\]

\[
F_1 + F_2 = 30.6 i + 22.08 j
\]

\[
\|F_1 + F_2\| = \sqrt{(30.6)^2 + (22.08)^2} = 37.74
\]

\[
\tan \theta = \frac{22.08}{30.61} = 35.8°
\]
Barge B is pulled by two tugboats A and C. At a given instant the tension in cable AB is 4500-lb and the tension in cable BC is 2000-lb. Determine the magnitude and direction of the resultant of the two forces applied to B at that instant.

The resultant force is the sum of the forces.

\[ A = 4500 \cos 30^\circ \mathbf{i} + 4500 \sin 30^\circ \mathbf{j} = 3897.11 \mathbf{i} + 2250 \mathbf{j} \]
\[ B = 2000 \cos -45^\circ \mathbf{i} + 2000 \sin -45^\circ \mathbf{j} = 1414.21 \mathbf{i} - 1414.21 \mathbf{j} \]
\[ A + B = 5311.32 \mathbf{i} + 835.79 \mathbf{j} \]
\[ ||A + B|| = \sqrt{5311.32^2 + 835.79^2} = 5376.68 \text{ lbs} \]
\[ \tan \theta = \frac{835.79}{5311.32} = 0.1560 \]
\[ \theta = 8.9^\circ \]
An airplane is flying on a bearing N60°W at 500 mph. The wind is blowing bearing N70°E at 20 mph. Find the actual ground speed and direction of the airplane.

\[ \vec{a} = 500 \cos 150^\circ \hat{i} + 500 \sin 150^\circ \hat{j} \]
\[ \vec{w} = 20 \cos 20^\circ \hat{i} + 20 \sin 20^\circ \hat{j} \]

Airplane in component form: \(-433.013\hat{i} + 250\hat{j}\)
Wind in component form: \(18.794\hat{i} + 6.84\hat{j}\)

Add to find resultant vector: \(-414.219\hat{i} + 256.84\hat{j}\)
\[ \| \vec{a} + \vec{w} \| = \sqrt{(-414.219)^2 + (256.84)^2} \approx 487.23 \text{ mph} \]

Change back to magnitude/direction:
\[ \tan \theta = \frac{256.84}{-414.219} \]
\[ \theta = -31.8^\circ \]
\[ \Theta = 180^\circ - 31.8^\circ \]
\[ N58.2^\circ \text{ W} \]

---

A ship is heading due East at 15 mph. The current is flowing NW at 3 mph. Find the actual bearing and speed of the ship.
Assignment:
Lesson 6.6
39-45 odd,
61-73 odd,
79-85 odd

Worksheet for extra resultant problems is posted on my moodle website.