Lesson 3.3
Properties of Logs

The Product Rule
Properties of exponents correspond to properties of logarithms. For example, when we multiply with the same base, we add exponents.

\[ b^m \cdot b^n = b^{m+n} \]

This property of exponents, coupled with an awareness that a logarithm is an exponent, suggests the following property.

**The Product Rule**
Let \( b, M, \) and \( N \) be positive real numbers with \( b \neq 1 \).

\[ \log_b(MN) = \log_b M + \log_b N \]

The logarithm of a product is the sum of the logarithms.
The Quotient Rule
When we divide with the same base, we subtract exponents.
\[ \frac{b^m}{b^n} = b^{m-n} \]
This property suggests the following property of logarithms.

The Quotient Rule
Let \( b, M, \) and \( N \) be positive real numbers with \( b \neq 1 \).
\[ \log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N \]
The logarithm of a quotient is the difference of the logarithms.

The Power Rule
When an exponential expression is raised to a power, we multiply exponents.
\[ (b^m)^n = b^{mn} \]
This property suggests the following property of logarithms.

The Power Rule
Let \( b \) and \( M \) be positive real numbers with \( b \neq 1 \), and let \( p \) be any real number.
\[ \log_b M^p = p \log_b M \]
The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.
When we use the product rule, quotient rule or power rule to write a single logarithm as the sum or difference of two logarithms we say that we are expanding a logarithmic expression.

Use the properties of logs to expand

\[
\begin{align*}
\log_6 (7 \cdot 11) &= \log_6 7 + \log_6 11 \\
\log_6 3^9 &= 9 \log_6 3 \\
\ln \frac{e^5}{11} &= \ln e^5 - \ln 11 \\
&= 5 \ln e - \ln 11 \\
&= 5 - \ln 11 \\
\log_8 \frac{23}{x} &= \log_8 23 - \log_8 x \\
\log(100x) &= \log 100 + \log x \\
&= 2 + \log x \\
\log(x+4)^2 &= 2 \log(x+4)
\end{align*}
\]

Expand the following:

\[
\begin{align*}
\log_2 (8xy^4) &= \log_2 8 + \log_2 x + 4 \log_2 y \\
&= 3 + \log_2 x + 4 \log_2 y \\
\ln \frac{\sqrt{x^2 + 5}}{x} &= \ln \left(\sqrt{x^2 + 5}\right) - \ln x \\
&= \frac{1}{2} \ln (x^2 + 5) - \ln x
\end{align*}
\]
To condense a logarithmic expression, we write the sum or difference of two or more logarithmic expressions as a single logarithmic expression. We use the properties of logarithms to do so.

**Properties for Condensing Logarithmic Expressions**

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b(MN)$ **Product rule**
2. $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$ **Quotient rule**
3. $p \log_b M = \log_b M^p$ **Power rule**

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Condense the following:

5 ln$x$ - 2 ln$(xy)$

$$\ln x^5 - \ln(xy)^2$$

ln$(x^5)$ = $\ln \frac{x^5}{(xy)^2}$

log25 + log4

$$\log (254) = \log 100 = 2$$

log$(7x+6)$ - log$x$

$$\log \frac{7x+6}{x}$$

4log$_3$x - 2log$_3$6 - $\frac{1}{2}$log$_3$y

$$\log_3 x^4 - (\log_3 6^2 + \frac{1}{2}\log_3 y)$$

$\log_3 x^4 - \log_3 (36\sqrt{y})$
Solve without using a calculator

\[
\log 85 - \log 17 + \frac{1}{2} \log 400
\]

\[
\log \frac{85}{17} + \log 400^{\frac{1}{2}}
\]

\[
\log 5 + \log 20
\]

\[
\log (5 \cdot 20)
\]

\[
\log 100 = 2
\]

We can log both sides of an equation or remove the logs (de-log) from each side of an equation to help solve the equation.

\[
\ln(x+3) = \ln 17
\]

\[
x+3 = 17 \quad x=14
\]

\[
\log x = \log (6-x)
\]

\[
x=6-x \quad 2x=6 \quad x=3
\]
\[ \log_3{17} = x \]

\[ 3^x = 17 \quad \text{(left right left)} \]
\[ \log_3{3^x} = \log_17 \quad \text{(log both sides)} \]
\[ x \cdot \log_3 = \log_17 \quad \text{(power rule)} \]
\[ x = \frac{\log_17}{\log_3} \quad \text{(divide both sides by log}_3) \]
\[ x = 2.57 \]

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Use this same method to solve:

\[ \log_6{10} = x \]

\[ 6^x = 10 \]
\[ \log_6{6^x} = \log_6{10} \]
\[ x \cdot \log_6 = \frac{\log_6{10}}{\log_6} \quad \text{\(x = \frac{\log_6{10}}{\log_6} \)} \]
\[ x = 1.29 \]

Do you see a shortcut?
Change of Base

\[ \log_b M = \frac{\log_a M}{\log_a b} \]  
(Use 10 or e for a)

\[ \log_4 15 \quad \log_{1/2} 2 \]

\[ \frac{\log 15}{\log 4} \quad \frac{\log 2}{\log 1/2} \]

Now let's graph

\[ f(x) = \log_4 x \]
\[ f(x) = \frac{\log x}{\log 4} \]

\[ f(x) = \log_{1/4} x \]
\[ f(x) = \frac{\log x}{\log 1/4} \]

\[ = \frac{\log x}{\log 1 - \log 4} \]
\[ = \frac{\log x}{-\log 4} \]
Describe the transformations of the following log functions:

\[ f(x) = \log_2(x+3) \]

\[ f(x) = \log_{\frac{1}{2}}(x) \]

\[ f(x) = \log_{\frac{1}{2}}(x+3) \]

ASSIGNMENT
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12, 14, 24, 27, 30, 35
42-60 * 3, 69-77 odd
81, 82, 89-97 odd