Complex Numbers

Here's another math problem I can't figure out. What's 9 x 4?

Don't that's a trick one. You have to use calculus and imaginary numbers for this?

MAGNIFICENT NUMBERS?

You know, eleven, twelve, and all that's a little confusing at first. How did you learn all this? I've never even gone to school. Instinct. Tigers are born with it.

Life before complex numbers.
Complex Numbers

\[ a + bi \]

(Standard Form)

a is real part, b is imaginary part

\[ a + bi = c + di \] iff \( a = c \) and \( b = d \)

Remember:

\[ i^2 = -1 \] and \( \sqrt{-1} = i \)

-4 + 6i

Real part -4
Imaginary part 6
Complex number

0 + 2i

Real part 0
Imaginary part 2
Pure imaginary

3 + 0i

Real part 3
Imaginary part 0
Real number

Remember, all three examples are complex numbers.
Operations with complex numbers:

add: $(3+2i)+(5-i) = 8 + i$  \hspace{1cm} (0+0i) add. identity

subt: $(5-2i)-(7-6i) = -2 + 4i$

mult: $(2-i)(4+5i) = 8 + 10i - 4i - 5i^2 = \frac{12 - 36i}{13 + 6i}$  \hspace{1cm} $(1 + 0i)$ mult. identity

div: \[
\frac{(4-2i)}{(6+6i)} = \frac{24 - 24i - 12i + 12i^2}{36 - 36i + 36i - 36i^2} = \frac{12 - 36i}{36} = \frac{1}{3} - \frac{3}{2}i
\]
use complex conjugate

\[
\frac{12 - 36i}{36} = \frac{1}{3} - \frac{3}{2}i
\]

\[
\frac{1}{3} - \frac{3}{2}i = \frac{1}{6} - \frac{1}{2}i
\]

Simplify

\[
\sqrt{25} = 5 \hspace{1cm} \sqrt{-25} = 5i
\]

\[
\sqrt{48} = 4\sqrt{3} \hspace{1cm} \sqrt{-48} = 4i\sqrt{3}
\]

\[
\sqrt{16 \cdot 3} = 4\sqrt{3} \hspace{1cm} \sqrt{17} = \sqrt{17}
\]

\[
\sqrt{-17} = i\sqrt{17}
\]
Perform the indicated operations and write the result in standard form.

\[
\frac{\sqrt{-27} + \sqrt{-48}}{\sqrt{9} \cdot 3 \cdot -1 + \sqrt{16} \cdot 3 \cdot -1} = 3i \sqrt{3} + 4i \sqrt{3} = 7i \sqrt{3}
\]

\[
(\frac{-2 + \sqrt{-3}}{2})^2 = 4 - 2i \sqrt{3} - 2i \sqrt{3} + 3i^2 = 1 - 4i \sqrt{3}
\]

\[
\frac{-14 + \sqrt{-12}}{2} = \frac{-14 + 2i \sqrt{3}}{2} = \frac{-14 + 2i \sqrt{3}}{2} = -7 + i \sqrt{3}
\]

Complex solutions in quadratic equations

\[3x^2 + 4x + 2 = 0 \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Discriminant: \(b^2 - 4ac\)

\(b^2 - 4ac > 0\) \quad 2 complex solutions - both real

\(b^2 - 4ac = 0\) \quad 1 complex solution - real

\(b^2 - 4ac < 0\) \quad 2 complex solutions - complex conjugates.
Solve using the quadratic formula, write answer in standard form

\[ 3x^2 - 2x + 4 = 0 \]

\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \frac{2 \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)} \]

\[ \frac{2 \pm \sqrt{-44}}{6} \]

\[ \frac{2 \pm 2i\sqrt{11}}{6} \]

\[ \frac{\frac{2}{3} \pm \frac{2i\sqrt{11}}{3}}{6} \]

Assignment:
Lesson 2.1

5, 9, 13, 19, 25, 27, 31, 35, 39, 41, 47, 49, 51, 55, 56, 57, 61