Lesson 3.2
Logarithmic Functions

Objectives
1. Change from logarithmic to exponential form.
2. Change from exponential to logarithmic form.
3. Evaluate logarithms.
4. Use basic logarithmic properties.
5. Graph logarithmic functions.
6. Find the domain of a logarithmic function.
7. Use common logarithms.
8. Use natural logarithms.

Study Tip
The discussion that follows is based on our work with inverse functions in Section 1.8. Here is a summary of what you should already know about functions and their inverses.

1. Only one-to-one functions have inverses that are functions. A function, $f$, has an inverse function, $f^{-1}$, if there is no horizontal line that intersects the graph of $f$ at more than one point.
2. If a function is one-to-one, its inverse function can be found by interchanging $x$ and $y$ in the function's equation and solving for $y$.
3. If $f(a) = b$, then $f^{-1}(b) = a$. The domain of $f$ is the range of $f^{-1}$. The range of $f$ is the domain of $f^{-1}$.
4. $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
5. The graph of $f^{-1}$ is the reflection of the graph of $f$ about the line $y = x$.

$y = b^x$  (exponential function)
find the inverse
$x = b^y$
now, how do we solve for $y$?
Definition of the Logarithmic Function
For $x > 0$ and $b > 0$, $b \neq 1$,

$$y = \log_b x$$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the **logarithmic function with base $b$.**

<table>
<thead>
<tr>
<th>Logarithmic Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \log_b x$</td>
<td>$x = b^y$</td>
</tr>
</tbody>
</table>

**Location of Base and Exponent in Exponential and Logarithmic Forms**

Logarithmic Form: $y = \log_b x$
Exponential Form: $b^y = x$

I use "left right left"

Write each equation in its equivalent exponential form

$$\log_5 x = 2 \quad \log_b 64 = 3 \quad \log_3 7 = y$$

$$5^2 = x \quad b^3 = 64 \quad 3^y = 7$$

Write each equation in its equivalent logarithmic form

$$12^2 = x \quad b^3 = 8 \quad e^y = 9$$

$$\log_{12} x = 2 \quad \log_b 8 = 3 \quad \log_e 9 = y$$
Simplify the following using the definition of logs (no calc)

\[
\begin{align*}
\log_{10} 0.1 &= x \quad 10^x = 0.1 \quad x = -1 \\
10^x &= 10^0 \quad x = 0 \\
10^x &= 10^3 \quad x = 3 \\
\log_{10} 1000 &= x \\
\log_2 8 &= x \quad 2^x = 8 \quad x = 3 \\
2^x &= 2^3 \quad x = 3 \\
\log_3 \sqrt[3]{3} &= x \quad 3^x = \sqrt[3]{3} \\
3^x &= 3^{1/3} \quad x = \frac{1}{3} \\
\log_5 1/25 &= x \quad 5^x = \frac{1}{5} \\
5^x &= 5^{-2} \quad x = -2 \\
\log_4 1 &= x \quad 4^x = 1 \quad x = 0 \\
\log_2 \sqrt[6]{2} &= x \quad 2^x = \sqrt[6]{2} \\
2^x &= 2^{1/6} \quad x = \frac{1}{6} \\
\log_8 9 &= x \\
8^x &= 9 \quad q^{2x} = 9 \\
(q^2)^x &= 9 \quad 2^x = \frac{1}{3} \\
(q^2)^x &= 9 \quad 2^x = \frac{1}{3} \\
\log_{log_b x} x &= x \quad \log_{log_b x} x = \log_{log_b x}
\end{align*}
\]

**Basic Properties of Logarithms**

For \(0 < b \neq 1, x > 0\), and any real number \(y\),

- \(\log_b 1 = 0 \) because \(b^0 = 1\).
- \(\log_b b = 1 \) because \(b^1 = b\).
- \(\log_b b^y = y \) because \(b^y = b^y\).
- \(b^{\log_b x} = x \) because \(\log_b x = \log_{log_b x}\).
Graphs of Exponential and Logarithmic Functions

\[ f(x) = 2^x \quad \text{g}(x) = \log_2 x \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 2^x )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{1}{2} )</th>
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<th>2</th>
<th>4</th>
<th>8</th>
</tr>
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<tbody>
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<td>( \log_2 x )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Domain \( (-\infty, \infty) \)
Range \( (0, \infty) \)
H.A. \( y = 0 \)

Domain \( (0, \infty) \)
Range \( (-\infty, \infty) \)
V.A. \( x = 0 \)

**common log** \( \log_{10} \)

\( y_1 = \log x \)
Zoom 6

**natural log** \( \log_e \)

*Watch for calculator failure - range of \( \log_{10} x \) is all reals*
Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: $(0, \infty)$. The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$.

2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass through the point $(1, 0)$ because $f(1) = \log_b 1 = 0$. The $x$-intercept is 1. There is no $y$-intercept.

3. If $b > 1$, $f(x) = \log_b x$ has a graph that goes up to the right and is an increasing function.

4. If $0 < b < 1$, $f(x) = \log_b x$ has a graph that goes down to the right and is a decreasing function.

5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the $y$-axis. The $y$-axis, or $x = 0$, is a vertical asymptote. As $x \to 0^+$, $\log_b x \to -\infty$ or $\infty$. 

$f(x) = e^x$

$f(x) = \ln x$
Transformations of log functions. Find the domain of each.

\[ g(x) = \ln(x + 2) \quad h(x) = \ln(3 - x) \quad \ln(x-3) \]

\[ g(x) = 3 \log x \quad h(x) = 1 + \log x \]

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
</table>
| Vertical translation         | \( g(x) = \log_b x + c \)  \( g(x) = \log_b x - c \) | - Shifts the graph of \( f(x) = \log_b x \) upward \( c \) units.  
- Shifts the graph of \( f(x) = \log_b x \) downward \( c \) units. |
| Horizontal translation       | \( g(x) = \log_b(x + c) \)  \( g(x) = \log_b(x - c) \) | - Shifts the graph of \( f(x) = \log_b x \) to the left \( c \) units.  
- Vertical asymptote: \( x = -c \)  
- Shifts the graph of \( f(x) = \log_b x \) to the right \( c \) units.  
- Vertical asymptote: \( x = c \) |
| Reflection                   | \( g(x) = -\log_b x \)  \( g(x) = \log_b(-x) \) | - Reflects the graph of \( f(x) = \log_b x \) about the \( x \)-axis.  
- Reflects the graph of \( f(x) = \log_b x \) about the \( y \)-axis. |
| Vertical stretching or shrinking | \( g(x) = c \log_b x \) | - Vertically stretches the graph of \( f(x) = \log_b x \) if \( c > 1 \).  
- Vertically shrinks the graph of \( f(x) = \log_b x \) if \( 0 < c < 1 \). |
| Horizontal stretching or shrinking | \( g(x) = \log_b(cx) \) | - Horizontally shrinks the graph of \( f(x) = \log_b x \) if \( c > 1 \).  
- Horizontally stretches the graph of \( f(x) = \log_b x \) if \( 0 < c < 1 \). |
Assignment
Lesson 3.2
3-42 mult 3, 47-52, 53, 57, 75-99 mult 3

Left Right Left

Left Right Left (you tube)
Evaluate without using a calculator

\[ \log_2(\log_3 81) \]

\[ \log_3(\log_2 \frac{1}{8}) \]

Measuring sound using decibels

\[ \beta = 10 \log \left( \frac{I}{I_0} \right) \]

\[ I_0 = 10^{-12} \]

\[ I_0 = \text{hearing threshold} \]

### Table 3.17 Approximate Intensities of Selected Sounds

<table>
<thead>
<tr>
<th>Sound</th>
<th>Intensity (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearing threshold</td>
<td>(10^{-12})</td>
</tr>
<tr>
<td>Soft whisper at 5 m</td>
<td>(10^{-11})</td>
</tr>
<tr>
<td>City traffic</td>
<td>(10^{-5})</td>
</tr>
<tr>
<td>Subway train</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>Pain threshold</td>
<td>(10^0)</td>
</tr>
<tr>
<td>Jet at takeoff</td>
<td>(10^3)</td>
</tr>
</tbody>
</table>


How loud is a train inside a subway tunnel?
Assignment
Lesson 3.2 Day 2
101-111 odd, 115,
117, 119 a,b, 139-144