Lesson 6.3 Polar Coordinates
How would I measure the distance from my house to the school?
How would I measure the distance from here to Target field?

- miles
- blocks
- inches
- minutes
- meters
- feet
- light years
- football fields

We use different measurements for different situations. Same for points...

Polar coordinate system

Many physical systems—such as those concerned with bodies moving around a central point or with phenomena originating from a central point—are simpler and more intuitive to model using polar coordinates. The initial motivation for the introduction of the polar system was the study of circular and orbital motion.

Polar coordinates are used often in navigation, as the destination or direction of travel can be given as an angle and distance from the object being considered. For instance, aircraft use a slightly modified version of the polar coordinates for navigation. In this system, the one generally used for any sort of navigation, the 0° ray is generally called heading 360, and the angles continue in a clockwise direction, rather than counterclockwise, as in the mathematical system.
Rectangular coordinate system \((x, y)\)

\[
A = [3, \frac{\pi}{4}]
\]
\[
B = [3, \frac{3\pi}{4}]
\]
\[
C = [2, \frac{7\pi}{6}]
\]
\[
D = [1, \frac{5\pi}{3}]
\]

Polar coordinate system \([r, \theta]\)

\[
[r, \theta] \quad \text{(directed distance)}
\]
\[
\theta = \text{directed angle}
\]

\[
\begin{bmatrix}
3, \frac{2\pi}{3} \\
-2, \frac{\pi}{3} \\
-4, -\frac{\pi}{4}
\end{bmatrix}
\]
Plotting Points on a Polar Grid

\[
\begin{bmatrix}
2, \frac{\pi}{3} \\
2, -\frac{5\pi}{3} \\
-2, \frac{4\pi}{3} \\
-2, -\frac{2\pi}{3}
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1, \frac{3\pi}{4} \\
-1, -\frac{5\pi}{4} \\
1, \frac{7\pi}{4} \\
1, -\frac{7\pi}{4}
\end{bmatrix}
\]

Multiple Representations of Points

If \( n \) is any integer, the point \( (r, \theta) \) can be represented as

\[
[r, \theta] = (r, \theta + 2n\pi) \quad \text{or} \quad (r, \theta) = (-r, \theta + 2n\pi).
\]

Find another representation of \[
\begin{bmatrix}
5, \frac{3\pi}{4}
\end{bmatrix}
\]
in which

- \( r \) is positive and \( 2\pi < \theta < 4\pi \)

- \( r \) is negative and \( 0 < \theta < 2\pi \)

- \( r \) is positive and \( -2\pi < \theta < 0 \)
Conversion - from polar coordinates to rectangular

\[ [r, \theta] = \text{polar form} \]
\[ (x, y) = \text{rectangular form} \]

\[
\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}
\]

\[
x = r \cdot \cos \theta \quad y = r \cdot \sin \theta
\]

Polar to rectangular

\[
\begin{bmatrix}
2, \frac{3\pi}{2} \\
(0, -2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
4, \frac{5\pi}{4} \\
(-2.83, -2.83)
\end{bmatrix}
\]

\[
x = r \cos \theta \quad y = r \sin \theta
\]

\[
x = 2 \cos \left(\frac{3\pi}{2}\right) \\
x = 2 \cdot 0 = 0
\]

\[
y = 2 \sin \left(\frac{3\pi}{2}\right) \\
y = 2 \cdot (-1) = -2
\]

\[
x = 4 \cos \left(\frac{5\pi}{4}\right) \\
x = 4 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -2\sqrt{2}
\]

\[
y = 4 \sin \left(\frac{5\pi}{4}\right) \\
y = 4 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -2\sqrt{2}
\]
**Conversion - from rectangular coordinates to polar**

\[ x^2 + y^2 = r^2 \]
\[ \tan \theta = \frac{y}{x} \]

**Rectangular to Polar form**

(-1, 1)
\[ (-1)^2 + (1)^2 = r^2 \]
\[ r = \sqrt{2} \]
\[ \tan \theta = \frac{1}{-1} \]
\[ \theta = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4} \]

(-3, 0)
\[ (-3)^2 + (0)^2 = r^2 \]
\[ r = 3 \]
\[ \tan \theta = 0 \]
\[ \theta = 0 \text{ or } \pi \]
Assignment: Lesson 6.3  1-47 odd, 48

Equation Conversion
Rectangular to Polar coordinates

A polar equation is an equation whose variables are r and θ. To convert a rectangular equation in x and y to a polar equation in r and θ, replace x with r cos θ and y with r sin θ.
Convert the rectangular equation to a polar equation.

\[ x + y = 5 \]

Convert the rectangular equation to a polar equation.

\[ (x + 1)^2 + y^2 = 1 \]
Polar to Rectangular coordinates.

When we convert an equation from polar to rectangular coordinates, our goal is to obtain an equation in which the variables are $x$ and $y$, rather than $r$ and $\theta$. We use one or more of the following equations.

\[ r^2 = x^2 + y^2 \quad r \cos \theta = x \quad r \sin \theta = y \quad \tan \theta = \frac{y}{x} \]

Convert each polar equation to a rectangular equation in $x$ and $y$.

\[ r = 5 \quad r = 3 \csc \theta \]
Convert the polar equation to a rectangular equation in x and y.

\[ r = -6 \cos \theta \quad \quad r = 4 \cos \theta - 6 \sin \theta \]

Convert each polar equation to a rectangular equation in x and y.

\[ \theta = \frac{\pi}{4} \quad \quad \theta = 4\pi \]
Assignment:
Lesson 6.3 Day 2
49-73 odd, 79, 81, 83, 84, 103-106