Find the following probabilities using a deck of cards

P(Choosing 6 cards and 2 are kings)

\[
\frac{\binom{4}{2} \cdot \binom{48}{4}}{\binom{52}{6}}
\]

P(Choosing 5 cards with 1 Ace, 1 King and 1 Queen)

\[
\frac{\binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{40}{2}}{\binom{52}{5}}
\]
The math team has 27 members consisting of 6 seniors, 9 juniors, 7 sophomores and 5 freshmen. Only 8 members are allowed on the state math team. What is the probability of forming a state math team with:

All junior members \[ \frac{\binom{9}{8}}{27 \binom{8}{8}} \]

Two sophomores and two freshmen \[ \frac{\binom{7}{2} \cdot \binom{5}{2} \cdot \binom{15}{4}}{27 \binom{8}{8}} \]

**Conditional Probabilities**

Suppose there is a 40% chance of rain tomorrow. If it rains, there is a 20% chance that all the rides at ValleyFair will be operating. If it doesn't rain, there is a 90% chance all the rides will be operating. What is the probability that all of the rides will be operating tomorrow?
Tree diagrams are often helpful when calculating probabilities.

Suppose there is a 40% chance of rain tomorrow. If it rains, there is a 20% chance that all the rides at ValleyFair will be operating. If it doesn’t rain, there is a 90% chance all the rides will be operating. What is the probability that all of the rides will be operating tomorrow?

\[
\begin{align*}
\text{rain} & : 0.40 \\
\text{no rain} & : 0.60 \\
\text{rides} & : 0.20 \\
\text{no rides} & : 0.80
\end{align*}
\]

\[
\begin{align*}
0.40 \times 0.20 & = 0.08 \\
0.40 \times 0.80 & = 0.32 \\
0.60 \times 0.90 & = 0.54 \\
0.60 \times 0.10 & = 0.06
\end{align*}
\]

\[0.08 + 0.54 = 0.62\]

Two cookie jars contain the following cookies:

Jar A - 2 choc chip
2 peanut butter

Jar B - 1 choc chip

What is the probability that you will choose a chocolate chip cookie?

\[
\begin{align*}
\text{Jar A} & : 0.5 \\
\text{CC} & : 0.25 \\
\text{P But} & : 0.25 \\
\text{Jar B} & : 0.5 \\
\text{CC} & : 1
\end{align*}
\]

\[0.125 + 0.125 + 0.5 = 0.75\]
Conditional Probability

\[ P(A | B) \]

read "probability of A given B"

\[ P(A | B) = \frac{P(A \text{ and } B)}{P(B)} \]

What is the probability that the cookie came from Jar A given that it was a chocolate chip cookie?

\[ P(\text{Jar A} | \text{choc chip}) = \frac{P(\text{Jar A and choc chip})}{P(\text{choc chip})} = \frac{.25}{.75} = \frac{1}{3} \]
What is the probability that it rained given that the rides were running at ValleyFair?

\[
P(\text{rain} | \text{rides}) = \frac{P(\text{rides and rain})}{P(\text{rides})}
\]

\[
\begin{array}{rcl}
\text{rain} .40 & \quad & .40 \times .20 = .08 \\
\text{no rain} .60 & \quad & .60 \times .80 = .48 \\
\end{array}
\]

\[
\frac{.08}{.62} = .13
\]

---

**Binomial Distributions**

**Sample space for tossing 4 coins**

<table>
<thead>
<tr>
<th>4 heads</th>
<th>3 heads</th>
<th>2 heads</th>
<th>1 head</th>
<th>0 heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHHH</td>
<td>HHHT</td>
<td>HHTT</td>
<td>HTTT</td>
<td>TTTT</td>
</tr>
<tr>
<td>HHTH</td>
<td>THHT</td>
<td>HTHT</td>
<td>THTT</td>
<td>TTHT</td>
</tr>
<tr>
<td>HTHH</td>
<td>HTTH</td>
<td>TTHH</td>
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<td>HTTH</td>
</tr>
<tr>
<td>THHH</td>
<td>THTH</td>
<td>THHT</td>
<td>TTTT</td>
<td>HTTH</td>
</tr>
</tbody>
</table>

Notice that the number of ways to get heads is the same as the fourth row of Pascal's Triangle.

\[
1 \quad 4 \quad 6 \quad 4 \quad 1
\]
Expand $(H + T)^4$

Find $P$(at least 3 heads)

1 $(H)^4 (T)^0$
4 $(H)^3 (T)^1$
6 $(H)^2 (T)^2$
4 $(H)^1 (T)^3$
1 $(H)^0 (T)^4$

1 way to get 4 heads
4 ways to get 3 heads

$5/16 = .3125$

---

**Binomial Distributions**

**A Binomial Experiment MUST**...

- contain a certain number of trials
- have ONLY two outcomes - success or failure
- trials must be identical and independent
  (outcome of one trial does not affect the outcome of another trial)
- probability of success = $p$
- probability of failure = $1-p$

**example:** flipping a coin 10 times
**EXAMPLE:** Suppose that the probability of you receiving an email each day is 60%, what is the probability that you will receive an email 5 out of the next 7 days.

**Success:** You DO receive an email \([p = 0.60]\)

**Failure:** You DO NOT receive an email \([p = 1 - 0.60 = 0.40]\)

**Number of Trials:** \(n = 7\) days

**Number of Success:** \(r = 5\) days that you receive emails

\[
P = \binom{7}{5}(0.60)^5(0.40)^2 = 21 \cdot (0.07776) \cdot 0.16 = 0.261 \text{ or } 26.1%\
\]


**EXAMPLE:** Suppose that the probability of you receiving an email each day is 60%, what is the probability that you will receive an email 3 OR 5 days out of the next 7 days.

**Success:** You DO receive an email \([p = 0.60]\)

**Failure:** You DO NOT receive an email \([p = 1 - 0.60 = 0.40]\)

**Number of Trials:** \(n = 7\) days

**Number of Success:** \(r = 5\) days that you receive emails or \(s = 3\) days that you receive emails

\[
P = \binom{7}{5}(0.60)^5(0.40)^2 + \binom{7}{3}(0.60)^3(0.40)^4
\]

\[
= 0.194 + 0.261 = 0.455 \text{ or } 45.5%\
\]

**Remember that “OR” always implies that you will add the two probabilities!!**
Suppose you roll 12 die. What is the probability that no more than 2 are sixes?

\[
\begin{align*}
12 \cdot \binom{12}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{10} &= \quad \\
12 \cdot \binom{12}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^{11} &= \quad \\
12 \cdot \binom{12}{0} \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^{12} &= \quad \\
\end{align*}
\]

\[0.677\]

Suppose that you take a 10 question true/false quiz. What is the probability that you get between 5 and 8 (inclusive) correct?

\[
\begin{align*}
10 \cdot \binom{10}{5} \cdot (0.5)^5 \cdot (0.5)^5 &= \\
10 \cdot \binom{10}{6} \cdot (0.5)^6 \cdot (0.5)^4 &= \\
10 \cdot \binom{10}{7} \cdot (0.5)^7 \cdot (0.5)^3 &= \\
10 \cdot \binom{10}{8} \cdot (0.5)^8 \cdot (0.5)^2 &= \quad \\
\end{align*}
\]

\[0.6123\]
Suppose a certain drug has a 70% success rate on patients. What is the probability that more than 20 out of 25 new patients are cured with the drug?

\[ _{25}C_{21} (0.7)^{21} (0.3)^4 + _{25}C_{22} (0.7)^{22} (0.3)^3 + _{25}C_{23} (0.7)^{23} (0.3)^2 + _{25}C_{24} (0.7)^{24} (0.3)^1 + _{25}C_{25} (0.7)^{25} (0.3)^0 \]

Using binomial pdf and binomial cdf in your graphing calculator.
Suppose a certain drug has a 70% success rate on patients. What is the probability that exactly 4 out of 25 new patients are cured with the drug?

\[ \text{binompdf}(25, .7, 4) \]

3.44 x 10^{-8}

Suppose a certain drug has a 70% success rate on patients. What is the probability that no more than 4 out of 25 new patients are cured with the drug?

\[ \text{binompdf}(25, .7, 4) \]

3.44 x 10^{-8}
Suppose a certain drug has a 70% success rate on patients. What is the probability that at least 4 out of 25 new patients are cured with the drug?

\[ 1 - \text{binomcdf}(25, 0.7, 3) \]

\[ 0.999999997 \]

Assignment:
Probability
Day 2
Worksheet